## ECE 341 Final Exam Solution

Name: $\qquad$
Time allowed: 2 hours
Total Points: 100
Points Scored: $\qquad$

## Problem No. 1 (10 points)

For each of the following statements, indicate whether the statement is TRUE or FALSE. Each correct answer carries $\underline{2}$ points. The answer for the last statement counts towards extra credit.
(a) Any logic function expressed in a sum-of-products form can be implemented by using NAND gates. TRUE
(b) Pipelining reduces the time needed to process a single instruction. FALSE
(c) Increasing the cache block size while keeping the total cache size constant can sometimes hurt performance. TRUE
(d) The cost per bit for DRAM is lower than the cost per bit for SRAM. TRUE
(e) When interrupts are used to synchronize data transfer between a processor and an I/O device, the processor needs to repeatedly monitor the status of I/O device. FALSE
(f) (Extra credit question) In port-mapped I/O, any machine instruction that can access memory can also be used to transfer data to an I/O device. FALSE

## Problem No. 2 (12 points)

Multiple possible answers are provided for each of the following questions. Only one answer is correct in each case. Mark the correct answer for each question. Each correct answer carries $\underline{4}$ points. The answer for the last question counts towards extra credit.
(a) Consider a $8 \mathrm{M} \times 64$ memory built by using $512 \mathrm{~K} \times 16$ memory chips. What is the total number of memory chips needed?
i. $\quad 16$
ii. 32
iii. 64
iv. 128
(b) A processor uses 44-bit virtual addresses with 4 kB pages. Which bits in the virtual address correspond to the "virtual page number" field?
i. The least significant 12 bits
ii. The most significant 12 bits
iii. The least significant 32 bits
iv. The most significant 32 bits
(c) You are required to implement a logic function $f=x y+y z$ in its minimal form. You can use any combination of 2-input logic gates. What is the minimum number of logic gates required to implement $f$ ?
i. 1 gate
ii. $\underline{2}$ gates
iii. 4 gates
(d) (Extra credit question) A non-pipelined 5-stage RISC processor running at 2.5 GHz is used to execute a program $P_{1}$ with 1 billion instructions. $50 \%$ of the instructions in $P_{1}$ are Load or Store instructions. Assume that the processor does not use a cache. All the instruction fetch and data read/write requests are served by main memory with a fixed latency of 6 clock cycles. How long does it take for the processor to complete the program $\mathrm{P}_{1}$ ?
i. 1 second
ii. 2 seconds
iii. 5 seconds

## Problem No. 3 (18 points)

Consider the following sequence of instructions being processed on the pipelined 5-stage RISC processor discussed in class:

Load R4, \#100(R2)
Add R5, R2, R3
Subtract R6, R4, R5
And R7, R2, R5
(a) (6 points) Identify all the data dependencies in the above instruction sequence. For each dependency, indicate the two instructions and the register that causes the dependency.

## Solution:

There are three data dependencies in this instruction sequence:
(i) Subtract instruction depends on Load instruction for register R4
(ii) Subtract instruction depends on Add instruction for register R5
(iii) And instruction depends on Add instruction for register R5
(b) ( 6 points) Assume that the pipeline does not use operand forwarding. Also assume that the only sources of pipeline stalls are the data hazards. Draw a diagram that represents instruction flow through the pipeline during each clock cycle. How long does it take for the instruction sequence to complete?

## Solution:

The following diagram shows the instruction flow through the pipeline. As shown in the diagram, the instruction sequence is completed in $\underline{11}$ cycles.

(c) (6 points) Now, assume that the pipeline uses operand forwarding. There are separate forwarding paths from the outputs of stage-3 and stage-4 to the input of stage-3. Draw a diagram that represents the flow of instructions through the pipeline during each clock cycle. Indicate operand forwarding by arrows.

## Solution:

The following diagram shows the instruction flow through the pipeline in the presence of operand forwarding:


## Problem No. 4 (12 points)

Consider the following instruction sequence being processed on the non-pipelined 5-stage RISC processor discussed in class:

Instruction 1: Load R4, \#100(R2)
Instruction 2: Add R5, R3, R4
Instruction 3: Store R5, \#200(R2)
The processor data path with all the control signals is shown in the following figure:

(a) (8 points) Write down the values of following control signals:
I. B_select during stage 3 of instruction processing (for each of the three instructions).
II. Y_select during stage-4 of instruction processing (for instructions 1 and 2 only).
III. RF_write during stage-5 of instruction processing (for each of the three instructions).

## Solution:

i. B_select has values of $\mathbf{1 , 0}$ and $\mathbf{1}$ for instructions 1,2 , and 3 , respectively.
ii. $\quad Y_{-}$select has values of $\mathbf{1}$ and $\mathbf{0}$ for instructions 1 and 2 , respectively.
iii. RF_write has values of $\mathbf{1}, \mathbf{1}$ and $\mathbf{0}$ for instructions $\mathbf{1}, 2$, and 3 , respectively.
(b) (4 points) Assume that the initial contents of registers R2, R3 and R4 are 5000, 200 and 300, respectively. Also assume that the initial contents of memory address 5100 are 400 . Write down the contents of inter-stage register $R Y$ after the completion of stage- 4 for instructions 1 and 2.

## Solution:

For instruction 1:
Contents of $R Y=$ Contents of memory address $(5000+100)=400$
The contents of register R4 become 400 after instruction 1.
For instruction 2:
Contents of $R Y=[R 3]+[R 4]=200+400=600$

## Problem No. 5 (18 points)

(a) (6 points) You are required to design a 2-bit synchronous counter by using a finite state machine. The counter has two external input signals $r$ and $x$, which dictate the operation of the counter as follows: (i) If $\boldsymbol{r}=\mathbf{0}, \boldsymbol{x}=\mathbf{0}$, the counter counts up, (ii) If $\boldsymbol{r}=\mathbf{0}, \boldsymbol{x}=\mathbf{1}$, the counter counts down, (iii) If $r=1$, the counter stops counting and retains its present value, irrespective of the value of $x$. The counter also has an output signal $z$, which is equal to 1 only if the present value of the counter is an odd number. Draw the state diagram for this state machine.

## Solution:


(b) (6 points) A 4-bit carry-lookahead adder (CLA) is used to add two numbers $X\left(x_{3} x_{2} x_{1} x_{0}\right)$ and $Y$ $\left(y_{3} y_{2} y_{1} y_{0}\right)$ with a external carry-in of $c_{0}$. Recall that the CLA computes the propagate functions ( $P_{3} P_{1} P_{2} P_{0}$ ) and the generate functions $\left(G_{3} G_{2} G_{1} G_{0}\right)$ at each bit position, and then uses them to compute the carry-out values $\left(\mathrm{c}_{4} \mathrm{C}_{3} \mathrm{C}_{2} \mathrm{C}_{1}\right)$ at each bit position.

Show the logic expressions for the following outputs: (i) $P_{3}$, (ii) $G_{3}$, and (iii) $c_{4}$.

## Solution:

i. $\quad P_{3}=x_{3}+y_{3}$
ii. $\quad G_{3}=x_{3} \cdot y_{3}$
iii. $\quad \mathrm{C}_{4}=\mathrm{G}_{3}+\mathrm{P}_{3} \mathrm{G}_{2}+\mathrm{P}_{3} \mathrm{P}_{2} \mathrm{G}_{1}+\mathrm{P}_{3} \mathrm{P}_{2} \mathrm{P}_{1} \mathrm{G}_{0}+\mathrm{P}_{3} \mathrm{P}_{2} \mathrm{P}_{1} \mathrm{P}_{0} \mathrm{C}_{0}$
(c) (6 points) The input waveforms for a positive edge-triggered JK flip flop are shown in the following figure. Assume that each waveform starts at time $=0$ and output Q of the flip-flop has a logic value of " 0 " at time $=0$. What is the logic value of output $Q$ at the following points of time: (i) 10 ns, (ii) 20 ns , (iii) 40 ns ?


## Solution:

i. At $t=10 \mathrm{~ns}, \underline{\mathbf{Q}=\mathbf{0}}$ ( Q retains its initial value until there is a positive clock edge)

iii. At $t=40 \mathrm{~ns}, \underline{\mathbf{Q}=\mathbf{0}}$ ( Q toggles its value because $\mathrm{J}=1, \mathrm{~K}=1$ and there is a positive clock edge at 40 ns )

## Problem No. 6 (12 points)

A 5-stage pipelined RISC processor $C_{I}$ running at 2.2 GHz is used to execute a program $P_{l}$. The instruction statistics for $P_{l}$ are as follows:

Branches: 20\%
Loads: 20\%
Stores: 10\%
Arithmetic Instructions: 50\%
Assume that the program $P_{l}$ has no data dependencies. $C_{l}$ uses a dynamic branch predictor and a branch target buffer to predict the branch instructions with a prediction accuracy of $80 \%$. The computation of actual branch outcomes is carried out in the "decode" stage. Also assume that $C_{l}$ uses a cache such that $\mathbf{1 0 0 \%}$ of the instruction fetches and $\boldsymbol{x} \%$ of the data accesses hit in the cache. The penalty to access the main memory for a cache miss is 10 cycles.
A customer requires that the processor $C_{l}$ must achieve a throughput of 2 billion instructions per second. Calculate the minimum value of $x$ (data hit rate in the cache) which satisfies this requirement?

## Solution:

Clock Rate " $R$ " $=2.2 \mathrm{GHz}=2.2 * 10^{9} \mathrm{~Hz}$
We need to consider stall cycles due to both the cache misses and branch mispredictions.

## For cache misses:

Since $100 \%$ of the instruction fetches hit in the cache, the only source of cache misses are the data accesses caused by Load and Store instructions.
Percentage of Load and Store instructions $=20 \%+10 \%=30 \%$
Percentage of data accesses that hit in the cache $=x \%$
Therefore, percentage of data accesses that miss in the cache $=(100-x) \%$
Cache miss penalty $=10$ cycles
Therefore, $\delta_{\text {miss }}=$ stall frequency $*$ stall penalty $=0.3 *((100-\mathrm{x}) / 100) * 10=3-0.03 \mathrm{x}$
For branch mispredictions:
Percentage of branch instructions $=20 \%$
Since branch computation is carried out in "Decode" stage (stage-2), stall penalty = 1 cycle
Branch prediction accuracy $=80 \%$
Therefore, percentage of branches that are predicted inaccurately $=20 \%$
$\delta_{\text {branch_penalty }}=$ stall frequency $*$ stall penalty $=(0.2)(0.2) * 1=0.04$
Adding the two different sources of stall cycles:
Total stall cycles per instruction $\delta=\delta_{\text {miss }}+\delta_{\text {branch_penalty }}=3-0.03 \mathrm{x}+0.04=3.04-0.03 \mathrm{x}$
Throughput " $\mathrm{P}_{\mathrm{p}}$ " $=\mathrm{R} /(1+\delta)=\left(2.2 * 10^{9}\right) /(1+3.04-0.03 \mathrm{x})=\left(2.2 * 10^{9}\right) /(4.04-0.03 \mathrm{x})$
Since the required throughput is 2 billion instructions per second:
$2 * 10^{9}=\left(2.2 * 10^{9}\right) /(4.04-0.03 \mathrm{x})$
$4.04-0.03 \mathrm{x}=2.2 / 2=1.1$
Solving for " x " in the above equation yields $\mathrm{x}=98$.
Therefore, the required minimum cache hit rate for data accesses is $\underline{\mathbf{9 8} \%}$

## Problem No. 7 (18 points)

(a) ( 6 points) A computer system uses 32 -bit memory addresses. It has a 128 K -byte 8 -way setassociative cache, with 64 bytes per cache block. Assume that the size of each memory word is 1 byte. Calculate the number of bits in each of the Tag, Set, and Word fields of the memory address.

## Solution:

Block size $=64$ bytes $=2^{6}$ bytes $=2^{6}$ words
Therefore, Number of bits in the Word field = 6
Cache size $=128$ K-byte $=2{ }^{17}$ bytes
Number of cache blocks per set $=8=2^{3}$
Number of sets $=$ Cache size $/($ Block size $*$ Number of blocks per set $)=2^{17} /\left(2^{6} * 2^{3}\right)=2^{8}$
Therefore, Number of bits in the Set field = 8
Total number of address bits $=32$
Therefore, Number of bits in the Tag field $=32-6-8=\mathbf{1 8}$
(b) (12 points) A processor has a small direct-mapped cache capable of holding four cache blocks. Each cache block consists of 32 words. The processor uses 12 -bit memory addresses. Assume that the initial tag values for each cache block are as follows:

| Block | Tag |
| :---: | :---: |
| 00 | 00110 |
| 01 | 00001 |
| 10 | 00000 |
| 11 | Invalid |

The processor reads data sequentially from the following addresses: $32,48,64,128$
For each of the above addresses, indicate whether the cache access will result in a hit or a miss.

## Solution:

Block size $=32$ words $=2^{5}$ words $\Rightarrow>$ No. of bits in the Word field $=5$
Number of cache blocks $=4=2^{2} \Rightarrow$ No. of bits in the Block field $=2$
Total number of address bits $=12 \Rightarrow$ No. of bits in the Tag field $=12-5-2=5$
For a given 12-bit address, the 5 most significant bits represent the Tag, the next 2 bits represent the Block, and the 5 least significant bits represent the Word.

Access \# 1:
Address $=(32)_{10}=(000000100000)_{2}$. For this address, Tag $=\mathbf{0 0 0 0 0}$, Block $=01$
Current tag for cache block $01=00001$ (initial tag value from table)
Address tag does not match the block tag $=>$ cache miss
After this access, Tag field for cache block 01 is set to 00000
Access \# 2:
Address $=(48)_{10}=(000000110000)_{2}$. For this address, Tag $=\mathbf{0 0 0 0 0}$, Block $=01$
Current tag for cache block $01=\mathbf{0 0 0 0 0}$ (set in access \# 1)
Address tag matches the block tag => cache hit

Access \# 3:
Address $=(64)_{10}=(000001000000)_{2}$. For this address, Tag $=\mathbf{0 0 0 0 0}$, Block $=10$
Current tag for cache block $10=\mathbf{0 0 0 0 0}$ (initial tag value from table)
Address tag matches the block tag $=>$ cache $\underline{\text { hit }}$
Access \# 4:
Address $=(128)_{10}=(000010000000)_{2}$. For this address, Tag $=\mathbf{0 0 0 0 1}$, Block $=00$
Current tag for cache block $00=\mathbf{0 0 1 1 0}$ (initial tag value from table)
Address tag does not match the block tag $=>$ cache miss

